The method of presentation here sketched not only dispenses with Hesse's normal form entirely, but is to my mind simpler and more appropriate than the one ordinarily given. Direction cosines can be introduced by means of (1) whenever they are wanted, but there are very few occasions where there is any advantage in using them, the formula

$$\cos\alpha_1\cos\alpha_2+\cos\beta_1\cos\beta_2+\cos\gamma_1\cos\gamma_2$$

for the cosine of the angle between two lines being one of the few formulæ which are a little simpler when direction cosines are used than when written in terms of direction ratios.

As an introduction to more advanced work the method here suggested has the advantage of being applicable to isotropic lines and planes as well as to others.

II. Concerning a Method of Constructing the Graph of an Equation in Which the Variables May Be Separated.

By E. L. Rees, University of Kentucky.

If an equation can be put in the form $\varphi(x) = \psi(y)$, *i. e.*, if the variables can be separated, the graph may be constructed in the following simple manner: Construct the graphs of the equations $y' = \varphi(x)$, $x' = \psi(y)$. Draw an ordinate of the first curve and an abscissa of the second curve equal to the ordinate of the first. The intersection of this ordinate and abscissa is a point on the required curve. In this manner any desired number of points may be rapidly constructed.

The following theorems[1] giving the properties of the graph as determined from those of the auxiliary curves are easily deduced. Indeed, the truth of most of them may be inferred from geometric intuition.

1. If x' and y' are both increasing or both decreasing functions in corresponding intervals the curve will rise in those intervals.

2. If one function increases and the other decreases the curve will fall in the corresponding intervals.

3. A vertical line of symmetry of the first or a horizontal line of symmetry of the second auxiliary curve is a line of symmetry of the required curve.

4. A turning point (or a multiple turning point) on either auxiliary curve in general corresponds to a turning point (or multiple turning point) on the required curve.

5. Two corresponding turning points give a node if both are maximum or both minimum, or a conjugate point if one is maximum and the other minimum.

6. A double turning po nt and a corresponding turning point give a cusp.

7. A triple turning point and a corresponding turning point if both are maximum or both minimum give a point of osculation.

[1] In the statement of these theorems it is to be understood that turning points and multiple turning points on the second curve are those points at which the tangent is vertical, while these terms as applied to the first curve have their usual meaning.

Other theorems of a similar nature could be added. These theorems may be illustrated by the following simple equations, the numbers in the parentheses indicating the theorems which the equations are intended to illustrate. $y^2 - 1 = x^3 - 3x$, $y^2 - 1 = x^3$(1, 2, 3, 4); $y^2 - 2 = x^3 - 3x$, $y^2 + 2 = x^3 - 3x$ (5); $y^2 = x^3$ (6); $y^2 = x^4 - x^6$ (7).

It is, of course, true that the advantages of this method of constructing graphs are greatest for the more complicated forms of equations.

This principle may also be applied in the construction of graphs of parametric equations. For, if the parametric equations are $x = \varphi(t)$, $y = \psi(t)$ we may use as our auxiliary equations $x = \varphi(y')$, $y = \psi(x')$ and proceed as before.

UNDERGRADUATE MATHEMATICS CLUBS.

Edited by R. C. Archibald, Brown University, Providence, R. I.

In so far as information is obtainable the December issue of this Monthly will contain a directory of undergraduate mathematics clubs which have been in operation during at least a part of 1917–18. Certain statistics, and general discussion of club interests, will be introduced also. It is hoped that those clubs which have either failed to discover themselves to the editor or neglected to reply to his repeated solicitations for information, will report to him as soon as possible and, in any case, not later than October 6.

CLUB ACTIVITIES.

Mathematics Club of Iowa State Teachers College, Cedar Falls, Iowa.

This club was organized in December, 1909, "to further the work in mathematics in the college and in the state." The officers for 1917–18 were: President, Ira S. Condit, head of the department of ma hematics; secretary, Irma Hemphill '18; executive committee, Professor Peter Luteyn and Grace Hillier '18, who prepare programs and decide on dates of meetings The programs between June, 1916, and March 1918, were as follows:

June 28, 1916: "Teaching factoring" by Professor Robert D. Dougherty.

July 12: "Standard tests" by Professor E. E. Watson, of Parsons College.

August 2: "The problem and the process of analysis in arithmetic" by Professor Condit.

August 16: "Ratio and proportion," by Professor Charles W. Wester of Iowa State College and Rodney W. Babcock, instructor in mathematics at the University of Wisconsin.

October 18: "The place of mathematics in education" by Professor Wester.

November 15: Report of Committee on Elimination, of Iowa State Teachers Association, by Ruth Smith '18 and Tracey Hodson '18.

April 24, 1917: "Methods of statistical study" by Professor Wester.

May 16: Conservation in mathematics—"Conservation of time and energy by the development of algebraic symbolism" by Irma Hemphill '18; "The